

# Spin Precession and Time-Reversal Symmetry Breaking in Quantum Transport of Electrons Through Mesoscopic Rings

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We consider the motion of electrons through a mesoscopic ring in the presence of spin-orbit interaction, Zeeman coupling, and magnetic flux. The coupling between the spin and the orbital degrees of freedom results in the geometric and the dynamical phases associated with a cyclic evolution of spin state. Using a non-adiabatic Aharonov-Anandan phase approach, we obtain the exact solution of the system and identify the geometric and the dynamical phases for the energy eigenstates. Spin precession of electrons encircling the ring can lead to various interference phenomena such as oscillating persistent current and conductance. We investigate the transport properties of the ring connected to current leads to explore the roles of the time-reversal symmetry and its breaking therein with the spin degree of freedom being fully taken into account. We derive an exact expression for the transmission probability through the ring. We point out that the time-reversal symmetry breaking due to Zeeman coupling can totally invalidate the picture that spin precession results in effective, spin-dependent Aharonov-Bohm flux for interfering electrons. Actually, such a picture is only valid in the Aharonov-Casher effect induced by spin-orbit interaction only. Unfortunately, this point has not been realized in prior works on the transmission probability in the presence of both SO interaction and Zeeman coupling. We carry out numerical computation to illustrate the joint effects of spin-orbit interaction, Zeeman coupling and magnetic flux. By examining the resonant tunneling of

electrons in the weak coupling limit, we establish a connection between the observable time-reversal symmetry breaking effects manifested by the persistent current and by the transmission probability. For a ring formed by two-dimensional electron gas, we propose an experiment in which the direction of the persistent current can be determined by the flux-dependence of the transmission probability. That experiment also serves to detect if the electron-electron interaction can qualitatively alter the electronic states.

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## I. INTRODUCTION

The Aharonov-Bohm effect leads to a number of remarkable interference phenomena in mesoscopic systems, especially in rings [1]. Based on the discovery of the geometric phases [2], including the adiabatic Berry phase [3] and the nonadiabatic Aharonov-Anandan (AA) phase [4], it has been predicted that analogous interference phenomena can be induced by the geometric phases which originate from the interplay between electrons' orbital and spin degrees of freedom. Such interplay can be produced by external electric and magnetic fields, which lead to Zeeman coupling and spin-orbit (SO) interaction respectively.

Loss *et al.* first studied the textured ring embedded in inhomogeneous magnetic field [5]. They found the inhomogeneity of the field results in a Berry phase, which can produce the persistent currents. The effects of this Berry phase on conductivity were then discussed [6]. It was further pointed out that the adiabatic condition is not necessary for the geometric phase to exist, and the AA phase in textured rings can produce the persistent currents as well [7].

On the other hand, the Aharonov-Casher (AC) effect [8] in mesoscopic systems has attracted much attention. Meir *et al.* showed for the first time that SO interaction in one-dimensional (1D) rings results in an effective magnetic flux [9]. Mathur and Stone then pointed out that observable phenomena induced by SO interaction are the manifestations of the AC effect in electronic systems [10]. These authors investigated the effects of SO interaction on the persistent-current paramagnetism and the quantum transport in disordered systems, and obtained specific reduction factors for harmonics in AB oscillations [9–11]. In case the AC flux is not random, it can lead to interference phenomena as AB flux. Mathur and Stone proposed an observation of the AC oscillation of the conductance on semiconductor samples [10]. Balatsky and Altshuler [12] and Choi [13] studied the persistent currents produced by the AC effect.

Inspired by the study on textured rings, the AC effect has also been analyzed in connection with the spin geometric phase. Aronov and Lyanda-Geller considered the spin evolution

in conducting rings, and found that SO interaction results in a spin-orbit Berry phase which plays an interesting role in the transmission probability of the rings [14]. In their models, there is a Zeeman coupling from uniform magnetic field, but the SO Berry phase can be caused by SO interaction alone. So they have indeed shown the existence of the Berry phase in the AC effect. Since SO interaction is usually not strong enough to guarantee the validity of adiabatic approximation, a nonadiabatic treatment of the problem is necessary. In Ref. [15], we demonstrated the existence of a nonadiabatic AA phase in the AC effect in 1D rings. We found the AC flux and local spin orientations of the electronic eigenstates are determined by a spin cyclic evolution. In particular, we showed the AC phase comprises both the AA and the dynamical phases which are acquired in the cyclic evolution, and the adiabatic limit of the AA phase is just the SO Berry phase. Based on this geometric phase approach for the AC effect, Oh and Ryu studied the persistent currents produced by the cylindrically symmetric SO interaction in 1D rings [16].

As is well known, SO interaction is time-reversal invariant while Zeeman coupling breaks the time-reversal symmetry (TRS). Many prior works have shown the significance of the TRS and its breaking with regard to various interference phenomena caused by AB flux and SO interaction. It is therefore worthwhile to investigate if the coexistence of SO interaction and Zeeman coupling can produce any new observable effect with the spin degree of freedom being fully taken into account. However, most of the previous studies have focused on the rings in the presence of Zeeman coupling or SO interaction only. In Ref. [17], we have demonstrated that the competition between Zeeman coupling and SO interaction can produce persistent currents through the TRS breaking in a many-electron ring with a complete set of current-carrying single-particle states. For the transport properties, Aronov and Lyanda-Geller [14] have derived a transmission probability for a conducting ring in the presence of both the SO interaction and the Zeeman coupling, by making use of the concept of Berry phase. Unfortunately, they failed to take into account correctly the different properties of SO interaction and Zeeman coupling under the time-reversal transformation. As a result, they have not realized that their picture of effective flux for the interference

of spin-polarized electrons is actually invalidated by the TRS-breaking Zeeman coupling. Furthermore, even if the Zeeman coupling is absent, their expression for the effective flux induced by the SO interaction is still not complete. So the transport properties of a ring in the presence of both SO interaction and Zeeman coupling have not been solved yet and the roles of TRS and its breaking therein need to be clarified.

In this paper, we will discuss the transport properties of a ring in the presence of both the SO interaction and the Zeeman coupling. We will explore the roles of the TRS and its breaking in the transport phenomena when the spin degree of freedom is taken into account explicitly. We will also show the connection between the observable TRS-breaking effects manifested by the persistent current and by the transmission probability. Throughout the discussion, we will emphasize the TRS by investigating how the TRS-breaking Zeeman coupling affects the thermodynamic and transport properties of the system. The paper is organized as follows. In Sec. II, we first solve the spin cyclic evolution and find the corresponding geometric and dynamical phases for the system. The electronic eigenstates of the closed ring are then derived. The geometric phase and the exact solution for the ring with only Zeeman coupling [6,7] or SO interaction [15,16] are shown to be the limit of zero electric field or the limit of zero magnetic field in our results. On the other hand, the SO Berry phase, first introduced by Aronov and Lyanda-Geller [14], is simply the adiabatic limit of the geometric AA phase here. In Sec. III, we first derive an exact expression for the transfer matrices of the two ring branches (arms) by introducing four auxiliary spin states, which exhibit the orbital quantum number dependence of the spin orientations in electronic eigenstates. Then we calculate the transmission probability of the ring. We show that the presence of Zeeman coupling makes the spin orientations in energy eigenstates depend on the spin and the orbital quantum numbers simultaneously. As a consequence, the effective spin-dependent flux description, which has been established for SO interaction only, becomes inadequate. That explains why the results in Ref. [14] are not correct. When the Zeeman coupling is absent, the derived transmission probability agrees with the relations obtained by Meir *et al.* for general spin-independent thermodynamic and transport properties. We finally

carry out some numerical calculations to illustrate the effects of SO interaction and Zeeman coupling. We find there is an interesting and observable correspondence between the TRS-breaking effects manifested by the transmission probability and by the persistent current. That correspondence, if experimentally verified or excluded in some specific ring, may serve to detect if the electron-electron interaction is of qualitative importance in determining electronic states. In Sec. IV, we conclude with a summary of our results.

## II. GEOMETRIC PHASE AND EXACT SOLUTION

The Hamiltonian for an electron in the electric field  $\mathbf{E} = -\nabla V$  and the magnetic field  $\mathbf{B} = \nabla \times \mathbf{A}$  is

$$H = \frac{1}{2m_e}(\mathbf{p} - \frac{e}{c}\mathbf{A})^2 + eV - \frac{e\hbar}{4m_e^2c^2}\boldsymbol{\sigma} \cdot \mathbf{E} \times (\mathbf{p} - \frac{e}{c}\mathbf{A}) - \frac{ge\hbar}{4m_e c}\boldsymbol{\sigma} \cdot \mathbf{B}. \quad (1)$$

We consider a ring that is effectively one-dimensional (1D) and the fields which are cylindrically symmetric, i.e.,  $\mathbf{E} = E(\cos \chi_1 \mathbf{e}_r - \sin \chi_1 \mathbf{e}_z)$ ,  $\mathbf{B} = B(\sin \chi_2 \mathbf{e}_r + \cos \chi_2 \mathbf{e}_z)$  in the cylindrical coordinate system. For the ring lying in the  $xy$  plane with its center at the origin, the Hamiltonian is given by

$$H = \frac{\hbar^2}{2m_e a^2}[-i\frac{\partial}{\partial \theta} + \phi + \alpha(\sin \chi_1 \sigma_r + \cos \chi_1 \sigma_z)]^2 + \frac{\hbar \omega_B}{2}(\sin \chi_2 \sigma_r + \cos \chi_2 \sigma_z), \quad (2)$$

with  $\sigma_r = \sigma_x \cos \theta + \sigma_y \sin \theta$ ,  $\alpha = -\frac{eaE}{4m_e c^2}$  and  $\omega_B = -\frac{geB}{2m_e c}$ , where  $a$  is the ring radius,  $\theta$  is the angular coordinate and  $\phi$  is the enclosed magnetic flux in unit of flux quantum. The eigenvalue equation of the system can be solved through a straightforward diagonalization, as presented in Ref. [17]. Here we adopt the geometric phase approach [15], in order to identify the geometric and the dynamical phases in current-carrying eigenstates, which are responsible for transporting electrons when the ring is connected to current leads. How the phases and the spin orientations jointly affect the transmission probability will be elaborated in the next section.

The cylindrical symmetry of the system leads to the conservation of total angular momentum  $-i\partial/\partial\theta + \frac{1}{2}\sigma_z$ , which means the eigenstates of Hamiltonian (2) are of the form

$\Psi_{n,\mu}(\theta) = \exp(in\theta)\tilde{\psi}_{n,\mu}(\theta)/\sqrt{2\pi}$ , in which  $\mu = \pm$ ,  $n$  are arbitrary integers, and the spin states are given by

$$\tilde{\psi}_{n,+}(\theta) = \begin{bmatrix} \cos \frac{\beta_n}{2} \\ e^{i\theta} \sin \frac{\beta_n}{2} \end{bmatrix}; \quad \tilde{\psi}_{n,-}(\theta) = \begin{bmatrix} \sin \frac{\beta_n}{2} \\ -e^{i\theta} \cos \frac{\beta_n}{2} \end{bmatrix}, \quad (3)$$

where  $\beta_n$  is  $\theta$ -independent. From  $\Psi_{n,\mu}^\dagger \sigma_i \Psi_{n,\mu}$  as a function of  $\theta$ , it is readily seen that the local spin orientations at  $\theta$  is in the direction of  $\mu(\cos \beta_n \mathbf{e}_z + \sin \beta_n \mathbf{e}_r)$ . The explicit expression for the spin tilt angle  $\beta_n$  can be obtained by introducing a cyclic evolution of spin state for electrons encircling the ring, as presented in Ref. [15]. The geometric and the dynamical phases associated with the spin precession can thereby be identified for all of the energy eigenstates to determine the whole energy spectrum. Such an approach has the advantage of explicitly exhibiting the geometric phase and the adiabatic criterion, which acquire their original meanings in time-dependent problems. The spin cyclic evolution is defined by a time-dependent Schrödinger equation

$$i\hbar \frac{\partial}{\partial t} \psi(t) = H_s(t) \psi(t), \quad (4)$$

for a spin- $\frac{1}{2}$  particle in a time-varying magnetic field, where  $H_s$  is given by

$$H_s(t) = \alpha\hbar\omega[\sin \chi_1 \cos(\omega t)\sigma_x + \sin \chi_1 \sin(\omega t)\sigma_y + \cos \chi_1 \sigma_z] \\ + \frac{1}{2}\hbar\omega_B[\sin \chi_2 \cos(\omega t)\sigma_x + \sin \chi_2 \sin(\omega t)\sigma_y + \cos \chi_2 \sigma_z]. \quad (5)$$

From the solution of the cyclic evolution governed by Eq. (4), we obtain for  $\Psi_{n,\mu}$  the spin tilt angle

$$\tan \beta_n = \frac{2\alpha\omega_n \sin \chi_1 + \omega_B \sin \chi_2}{2\alpha\omega_n \cos \chi_1 + \omega_B \cos \chi_2 - \omega_n}, \quad (6)$$

and the geometric and the dynamical phases  $\delta_{n,\mu}$  and  $\gamma_{n,\mu}$

$$\delta_{n,\mu} = -\pi(1 - \mu \cos \beta_n), \quad (7)$$

$$\gamma_{n,\mu} = -\mu\pi[2\alpha \cos(\beta_n - \chi_1) + \frac{\omega_B}{\omega_n} \cos(\beta_n - \chi_2)], \quad (8)$$

where  $\omega_n$  is given by  $\omega_0(n + \frac{1}{2} + \phi)$  with  $\omega_0 = \frac{\hbar}{ma^2}$ . Here the geometric AA phase  $\delta_{n,\mu}$  is the -1/2 of the solid angle subtended by a circuit traced on a sphere by the local spin orientation of  $\Psi_{n,\mu}$ . It is readily seen that the Zeeman coupling makes the spin orientations of electronic eigenstates depend on the orbital quantum number. The consequence of such interplay between the spin and the orbital degrees of freedom will be explored when we discuss the transport properties of the ring.

With use of  $\beta_n$ ,  $\delta_{n,\mu}$ , and  $\gamma_{n,\mu}$ , the eigenvalues  $E_{n,\mu}$  of  $\Psi_{n,\mu}$  is found to be

$$E_{n,\mu} = \frac{\hbar\omega_0}{2}(n + \phi)^2 + \frac{\hbar\omega_0}{2}(\alpha^2 - \alpha \cos \chi_1) - \frac{\hbar\omega_n}{2\pi}(\delta_{n,\mu} + \gamma_{n,\mu}). \quad (9)$$

The first term in the right side represents the energy from orbital motion, the second term the zero-point energy, while the third term comes from the spin precession originating from the interplay between the spin and the orbital degrees of freedom.

The exact solution derived above reduces to the various limits that have been obtained separately in literatures. The cylindrically symmetric textured ring, first studied under the adiabatic approximation [5] and then exactly solved in Refs. [6,7], corresponds to the  $E = 0$  limit. The AC effect induced by cylindrically symmetric SO interaction in the ring, first discussed for vertical field [15] and then investigated for more general field configurations [16], corresponds to the  $B = 0$  limit. On the other hand, with both nonzero  $E$  and  $B$ , the SO Berry phase, first introduced for a conducting ring [14], is simply the adiabatic limit of the AA phase.

Now we turn to the adiabatic limit of the exact solution. Since the original stationary Schrödinger equation is solved via the solution of the time-dependent problem, the adiabatic criterion can be easily deduced. Comparing  $\beta_n$  with the tilt angle of the effective magnetic field in  $H_s$ , we find the adiabatic criterion is

$$\omega_n / (2\alpha\omega_n \cos \chi_1 + \omega_B \cos \chi_2) \rightarrow 0. \quad (10)$$

For the textured ring with  $\alpha = 0$ , this condition states that the Zeeman frequency  $\omega_B$  must be much larger than the orbital frequency  $\omega_n$  unless  $\chi_2 = 0$  [6,7]. For the AC effect with



$\omega_B = 0$ , this condition requires that the dimensionless SO coefficient  $\alpha$  must be much larger than 1 unless  $\chi_1 = 0$  [15,16].

### III. TRANSMISSION PROBABILITY

In this section, we discuss the transport properties of the ring described by Hamiltonian (2). Now the ring is connected to external current leads, schematically illustrated in Fig. 1. We adopt the standard formulation developed in the study of quantum oscillations in 1D rings threaded by AB flux [18]. In the upper and the lower branches, the wave amplitudes at one end are related to the wave amplitudes at the other end by the transfer matrices as

$$\begin{bmatrix} \beta_2 \\ \beta'_2 \end{bmatrix} = \underline{t}_I \begin{bmatrix} \beta'_1 \\ \beta_1 \end{bmatrix}, \quad \begin{bmatrix} \gamma_1 \\ \gamma'_1 \end{bmatrix} = \underline{t}'_{II} \begin{bmatrix} \gamma'_2 \\ \gamma_2 \end{bmatrix},$$

where  $t_I$  and  $t'_{II}$  denote the transfer matrices of the upper and lower branches respectively, and they depend on the energy  $E$  of the incident wave. At the two junctions, the amplitudes of the three outgoing waves  $(\alpha', \beta', \gamma')$  are related to the amplitudes of the incoming waves  $(\alpha, \beta, \gamma)$  by

$$\begin{bmatrix} \alpha' \\ \beta' \\ \gamma' \end{bmatrix} = \begin{bmatrix} -(a+b) & \sqrt{\epsilon} & \sqrt{\epsilon} \\ \sqrt{\epsilon} & a & b \\ \sqrt{\epsilon} & b & a \end{bmatrix} \begin{bmatrix} \alpha \\ \beta \\ \gamma \end{bmatrix},$$

where  $a = \pm(\sqrt{1-2\epsilon}-1)/2$  and  $b = \pm(\sqrt{1-2\epsilon}+1)/2$  with  $0 \leq \epsilon \leq 1/2$ . When considering a wave incident from the right junction, we have  $\alpha_1^\dagger \alpha_1 = 1$  and  $\alpha_2 = 0$ . The amplitude of the transmitted wave is

$$\alpha'_2 = -\frac{\epsilon}{b^2}([b-a, 1] \otimes \sigma_0) \underline{t}_I \Pi^{-1} \left( \begin{bmatrix} b-a \\ -1 \end{bmatrix} \otimes \sigma_0 \right) \alpha_1, \quad (11)$$

with  $\Pi$  given by

$$\Pi = \frac{1}{b^2} \left( \begin{bmatrix} b^2 - a^2 & a \\ -a & 1 \end{bmatrix} \otimes \sigma_0 \right) \underline{t}'_{II} \left( \begin{bmatrix} b^2 - a^2 & a \\ -a & 1 \end{bmatrix} \otimes \sigma_0 \right) \underline{t}_I - \mathbb{1}, \quad (12)$$

where  $\sigma_0$  is the  $2 \times 2$  unit matrix in spin space. This formulation is in general applicable to the derivation of the transmission probability through any ring, provided the corresponding transfer matrices are known. Note that in the study of the ring only threaded by AB flux, electrons can be treated as spinless particles, so that all amplitudes are simply represented by complex numbers and matrix  $\sigma_0$  can be dropped. In this paper,  $\alpha_1, \alpha'_1, \dots$  have to be represented by two-component spinors and  $\underline{t}_I, \underline{t}'_{II}$  are  $4 \times 4$  matrices.

### A. Cylindrical symmetry and electronic states in quantum transport

To derive an explicit expression for the two transfer matrices, we first identify the electronic states in the ring by making use of its cylindrical symmetry. If we write  $\underline{t}_I, \underline{t}'_{II}$  in  $2 \times 2$  matrix form, then each matrix element is a  $2 \times 2$  matrix in spin space. We can easily conclude that the off-diagonal elements of  $\underline{t}_I, \underline{t}'_{II}$  are zero because of the conservation of  $-i\frac{\partial}{\partial\theta} + \frac{1}{2}\sigma_z$ , which indicates that in each branch any propagating wave with fixed energy can possess a well-defined momentum and pass each branch without reflection, as a result of the cylindrical symmetry of the external fields and the absence of scattering potential. So our task reduces to finding the four  $2 \times 2$  matrices which respectively relate  $\beta'_1$  with  $\beta_2, \beta_1$  with  $\beta'_2$ , for the upper branch, and relate  $\gamma'_2$  with  $\gamma_1, \gamma_2$  with  $\gamma'_1$ , for the lower branch. These four  $2 \times 2$  matrices are the four nonzero diagonal elements of  $\underline{t}_I, \underline{t}'_{II}$  directly.

The electrons' tunneling through the ring is carried out by the energy eigenstate of the ring connected to two ideal conductors. Consider an incident wave with wavevector  $k_F$ . The corresponding eigenenergy of the steady transport state is  $E_F = \hbar^2 k_F^2 / 2m$ . In the right conductor the electronic state is a superposition of the incident plane wave  $\alpha_1$  and the reflected plane wave  $\alpha'_1$ , while in the left conductor the propagating wave is just the transmitted plane wave  $\alpha'_2$ . The state inside the ring is a superposition of four wavefunctions of energy  $E_F$ . They actually determine the four non-zero matrix elements defined above for the two diagonal transfer matrices.

To find the four components of the electronic wave inside the ring, we first use the energy

expression

$$\begin{aligned} \frac{\hbar^2 k_F^2}{2m} = E_{n,\mu} = & \frac{\hbar\omega_0}{2}(n + \phi)^2 + \frac{\hbar\omega_0}{2}(\alpha^2 - \alpha \cos \chi_1) \\ & + \frac{\hbar\omega_n}{2}(1 - \mu \cos \beta_n) + \mu\alpha\hbar\omega_n \cos(\beta_n - \chi_1) + \frac{\mu\hbar\omega_B}{2} \cos(\beta_n - \chi_2). \end{aligned} \quad (13)$$

to find four solutions of  $n$ , which are positive  $n_{+,+}$  and negative  $n_{-,+}$  with  $\mu = +$ , and positive  $n_{+,-}$  and negative  $n_{-,-}$  with  $\mu = -$ . For arbitrary  $k_F$ , these quantum numbers are not integers in general. For each  $n_{\lambda,\mu}$ , we can obtain a wavefunction  $\Psi_{n_{\lambda,\mu}}$  which bears the same form as  $\Psi_{n,\mu}$  of the closed ring, but with  $n$  being substituted by  $n_{\lambda,\mu}$  and accordingly the spin tilt angle  $\beta_n$  being substituted by  $\beta_{n_{\lambda,\mu}}$  from Eq. (6). These four  $\Psi_{n_{\lambda,\mu}}$  are actually eigenstates of the Hamiltonian (2) at energy  $E_F$  but the periodic boundary condition  $\Psi_{n,\mu}(\theta) = \Psi_{n,\mu}(\theta + 2\pi)$  is resolved due to the connection with external conductors. The electronic wave inside the ring is a superposition of the four  $\Psi_{n_{\lambda,\mu}}$  by which the eight amplitudes  $\beta_1, \beta'_1, \dots$  can be represented. This is a natural conclusion from the steadiness of the electronic state which transports electrons at fixed energy  $E_F$  through the ring. With this understanding, we can derive the transfer matrices in terms of  $\Psi_{n_{\lambda,\mu}}$ .

## B. Transfer matrices represented by nonorthogonal spin states

As shown in Sec. II, the Zeeman coupling brings the dependence on orbital quantum number to spin orientations. As a result,  $\Psi_{n_{\lambda,+,+}}$  and  $\Psi_{n_{\lambda,-,-}}$ , which carry the clockwise ( $\lambda = -$ ) or the anticlockwise ( $\lambda = +$ ) wave, are of nonorthogonal spin states  $\tilde{\psi}_{n_{\lambda,-,-}}(\theta)$  and  $\tilde{\psi}_{n_{\lambda,+,+}}(\theta)$  unless in the absence of Zeeman coupling. To derive the transfer matrix associated with spin-polarized transport, it is crucial to distinguish the  $\mu = +$  from the  $\mu = -$  contribution for any wave propagating in fixed direction. For this purpose, we define four auxiliary spin states

$$\tilde{\eta}_{\lambda,\mu}(\theta) = \frac{1}{R_\lambda}(\tilde{\psi}_{n_{\lambda,\mu}}(\theta) - \tilde{\psi}_{n_{\lambda,-\mu,-\mu}}^\dagger(\theta)\tilde{\psi}_{n_{\lambda,\mu}}(\theta)\tilde{\psi}_{n_{\lambda,-\mu,-\mu}}(\theta)), \quad (14)$$

where  $R_\lambda = 1 - |\tilde{\psi}_{n_{\lambda,\mu}}^\dagger(\theta)\tilde{\psi}_{n_{\lambda,-\mu}}(\theta)|^2$ . It is easy to verify the relations of redefined orthogonality and completeness,

$$\tilde{\eta}_{\lambda,\mu}(\theta)^\dagger \tilde{\psi}_{n_{\lambda,\nu},\nu}(\theta) = \delta_{\mu\nu}, \quad (15)$$

and

$$\sum_{\mu} \tilde{\psi}_{n_{\lambda,\mu},\mu}(\theta) \tilde{\eta}_{\lambda,\mu}(\theta)^\dagger = \sigma_0. \quad (16)$$

In the upper branch, the wave propagating anticlockwisely consists of the two components  $\Psi_{n_{+,+},+}$  and  $\Psi_{n_{+,-},-}$ .  $\beta'_1$  and  $\beta_2$  can thereby be expressed as

$$\begin{aligned} \beta'_1 &= c_1 \Psi_{n_{+,+},+}(0) + c_2 \Psi_{n_{+,-},-}(0); \\ \beta_2 &= c_1 \Psi_{n_{+,+},+}(\pi) + c_2 \Psi_{n_{+,-},-}(\pi), \end{aligned} \quad (17)$$

where  $c_1$  and  $c_2$  are two specific constants. Using Eqs. (3) and (15), we obtain

$$\beta_2 = [e^{in_{+,+}\pi} \tilde{\psi}_{n_{+,+},+}(\pi) \tilde{\eta}_{+,+}^\dagger(0) + e^{in_{+,-}\pi} \tilde{\psi}_{n_{+,-},-}(\pi) \tilde{\eta}_{+,-}^\dagger(0)] \beta'_1, \quad (18)$$

and therefore find the  $2 \times 2$  matrix which is the first diagonal element of  $\underline{t}_I$ . The other three matrix elements in diagonal  $\underline{t}_I$  and  $\underline{t}_{II}$  can be derived in the same way. We finally obtain the two transfer matrices in the form of

$$\underline{t}_I = \begin{bmatrix} \sum_{\mu} e^{in_{+,+}\pi} \tilde{\psi}_{n_{+,+},\mu}(\pi) \tilde{\eta}_{+,+}^\dagger(0) & 0 \\ 0 & \sum_{\mu} e^{in_{+,-}\pi} \tilde{\psi}_{n_{+,-},\mu}(\pi) \tilde{\eta}_{+,-}^\dagger(0) \end{bmatrix}; \quad (19)$$

$$\underline{t}'_{II} = \begin{bmatrix} \sum_{\mu} e^{in_{+,+}\pi} \tilde{\psi}_{n_{+,+},\mu}(0) \tilde{\eta}_{+,+}^\dagger(\pi) & 0 \\ 0 & \sum_{\mu} e^{in_{+,-}\pi} \tilde{\psi}_{n_{+,-},\mu}(0) \tilde{\eta}_{+,-}^\dagger(\pi) \end{bmatrix}. \quad (20)$$

From Eq. (11), the transmission probability for unpolarized incident electrons is  $\langle \alpha_2'^\dagger \alpha_2' \rangle$  in which  $\langle \dots \rangle$  denotes an averaging over  $\alpha_1$  with fixed  $\alpha_1^\dagger \alpha_1 = 1$ . Explicitly, it is given by

$$T = \frac{1}{2} \sum_{ij} \left| \left[ -\frac{\epsilon}{b^2} ([b-a, 1] \otimes \sigma_0) \underline{t}_I \Pi^{-1} \left( \begin{bmatrix} b-a \\ -1 \end{bmatrix} \otimes \sigma_0 \right) \right]_{ij} \right|^2, \quad (21)$$

in which  $\underline{t}_I$ ,  $\underline{t}'_{II}$ , and  $\Pi$  are all known.

### C. Success and breakdown of effective flux description

In the absence of Zeeman coupling, the expression of the transmission probability can be greatly simplified and be explicitly related to the spin-independent transmission probability through the ring threaded by AB flux only. From Eq. (6), it is obvious that if  $\omega_B = 0$ ,  $\tilde{\psi}_{n_{\lambda,\mu}}$  are independent of  $n_{\lambda,\mu}$  defined in Eq. (13) and can be denoted by  $\tilde{\psi}_\mu$  with  $\tilde{\eta}_{\lambda,\mu} = \tilde{\psi}_\mu$ . Combining this fact with Eqs. (15), (16), and (17), we see that in the absence of Zeeman coupling, the electronic wave in the ring actually consists of two orthogonal amplitudes, which propagate coherently and independently, with their local spin states being given by  $\tilde{\psi}_\mu$ . We then turn to the phase shift for spin-polarized electrons. When  $k_F a$  is very large and quasiclassical approximation is therefore applicable, it is worthwhile to write  $n_{\lambda,\mu}$  in Eq. (13) as

$$n_{\lambda,\mu} = \lambda k_F a - \phi - \frac{1}{2}(1 - \mu \cos \chi_{n_{\lambda,\mu}}) - \mu \alpha \cos \chi_{n_{\lambda,\mu}}, \quad (22)$$

where the last three terms in the right side are  $\frac{1}{2\pi}$  of the AB phase, the spin AA, and the dynamical phases contributed by the SO interaction, respectively. In case the Zeeman coupling is absent, the last two terms give the  $1/2\pi$  of the AC phase,  $\Phi_{AC}^\mu/2\pi$  [15], and

$$n_{\lambda,\mu} = \lambda k_F a - \phi + \frac{1}{2\pi} \Phi_{AC}^\mu \quad (23)$$

becomes an exact relation without quasiclassical approximation. Since  $\Phi_{AC}^\mu$  is  $n$ -independent, Eq. (23) indicates that the effect of the SO interaction can be regarded as an AB effect of the effective flux  $-\Phi_{AC}^\mu/2\pi$  in unit of  $\Phi_0$  for the locally polarized electron gases with local spin states  $\tilde{\psi}_\mu$ .

We can derive for  $\omega_B = 0$  the transmitted amplitude

$$\alpha'_2(\phi, \alpha_1) = \sum_\mu [\tilde{\psi}_\mu^\dagger(0) \alpha_1] t(\phi - \frac{\Phi_{AC}^\mu}{2\pi}) \tilde{\psi}_\mu(\pi), \quad (24)$$

where  $t(\phi)$  is the transmitted amplitude for the ring threaded by magnetic flux  $\phi$  [18] with vanishing SO interaction. Eq. (24) indicates clearly that the real electronic wave in the ring

is a superposition of the two locally polarized waves, which enclose different effective fluxes and propagate independently. The transmission probability  $T_{AB,AC}$  is given by  $\alpha_2'^\dagger \alpha_2'$ :

$$T_{AB,AC}(\phi, \alpha_1) = \sum_{\mu} |\tilde{\psi}_{\mu}^{\dagger}(0) \alpha_1|^2 T_{AB}(\phi - \frac{\Phi_{AC}^{\mu}}{2\pi}), \quad (25)$$

where  $T_{AB}(\phi) = t^{\dagger}t$  is the transmission probability of the ring threaded by magnetic flux  $\phi$  with vanishing SO interaction. To see what happens for unpolarized incident wave, we average  $T_{AB,AC}$  over  $\alpha_1$  and obtain  $\bar{T}_{AB,AC} = \sum_{\mu} T_{AB}(\phi - \Phi_{AC}^{\mu}/2\pi)/2$ , which agrees with the relation predicted in Ref. [9] for general spin-independent thermodynamic and transport quantities.

In the competition with the SO interaction, the Zeeman coupling brings the  $n$ -dependence to the spin orientations of energy eigenstates. The  $n$ -dependent spin precession then results in the  $n$ -dependent spin phases. It is seen that in the presence of the Zeeman coupling, the last two terms in Eq. (22) are  $n$ -dependent and the effect of the spin phases can no longer be regarded as that from the effective flux which must be independent of the specific orbital quantum numbers of the states. It is thus quite clear that in the presence of Zeeman coupling, we can not use 1) the identification of the two polarized wave amplitudes which are orthogonal to each other in spin space and propagate independently, and 2) the description that the phase effect from the spin degree of freedom is effectively some AB effect of a spin-dependent flux. We want to point out that the above complexity due to Zeeman coupling has not been recognized in Ref. [14]. Consequently the transmission probability obtained therein was wrongly simplified by regarding the contribution from the spin degree of freedom as an AB effect of some  $\mu$ -dependent flux for polarized electrons. We also want to point out that even when the Zeeman coupling is absent and the picture of effective flux is applicable, only the geometric phase was included while the dynamical phase was ignored in the expression of the effective flux in Ref. [14].

### D. Persistent current direction exhibited in transmission probability

Numerical calculation has been carried out to illustrate some essential characteristics of the transmission probability derived here. We find that the respective effects of Zeeman coupling and SO interaction can be reflected by the resonance of the transmission probability in the weak coupling limit at small  $\epsilon$ . In particular, we can see an interesting correspondence between the TRS-breaking effects manifested by the transmission probability and by the persistent current.

We adopt the model of a InAs ring [14]. The Hamiltonian is of the form

$$H_{\text{InAs}} = \frac{1}{2m}(\mathbf{p} - \frac{e\mathbf{A}}{c})^2 + \hbar\kappa[\boldsymbol{\sigma} \times \mathbf{p}]_z - \frac{ge\hbar}{4mc}\boldsymbol{\sigma} \cdot \mathbf{B}, \quad (26)$$

where  $m = 0.023m_e$  is the effective mass,  $\hbar^2\kappa = 6.0 \times 10^{-10}\text{eVcm}$  is the SO coefficient and  $g = 15$ . Here the effective electric field is in the z-direction, hence  $\chi_1 = \pi/2$ . For the ring of radius  $a = 1\mu\text{m}$ , the dimensionless coefficient  $\alpha$  in Eq. (2) is found to be  $m\alpha\kappa = 1.8$  which is large enough to result in an AC phase of order unity [15]. The Fermi velocity  $v_F$  is approximately  $3 \times 10^7\text{cms}^{-1}$ , corresponding to  $|n_F| \approx 60$ .

The effective flux induced by SO interaction and its effect on the transmission probability can be clearly seen in Fig. 2 where  $T_{AB}$  and  $\bar{T}_{AB,AC}$  are plotted as functions of  $\phi$ . The magnitude of the AC phase can actually be approximately measured by a comparison between the  $\phi$ -coordinates of the transmission probabilities' peaks in the absence and in the presence of the SO interaction. The energy splitting due to Zeeman coupling is illustrated in Fig. 3. For  $\phi = 0$  and  $\omega_B = 0$ , since the Kramers degeneracy makes each two eigenstates of the closed ring have the same energy, at certain  $E_F$  the transmission probabilities in the two spin branches can reach their highest value 1 simultaneously, thereby making  $\bar{T}_{AB,AC} = 1$ . After the Zeeman coupling is turned on, the resulted energy splitting destroys the simultaneous happenings of the resonances in the two spin branches and we see the maximum values of  $T$  decrease with the strength of Zeeman coupling appreciably.

With  $\epsilon$  being even smaller, the energy dependence of the transmission probability manifests interesting TRS-breaking effect, which also has its corresponding observability in per-

sistent current. In Ref. [17], it has been demonstrated that in the presence of SO interaction, the TRS-breaking mechanism due to Zeeman coupling is intrinsically different from that due to AB flux. As the corresponding observable effect, it has been found that the direction of the persistent current induced by Zeeman coupling changes periodically with the particle number  $N$  with the periodicity  $\Delta N = 2$  while the direction of the persistent current induced by AB flux never changes with the particle number. The dependence of the current direction on the particle number is actually the dependence on Fermi energy. Such energy dependence of the current direction, an equilibrium phenomenon as it is, can actually be manifested in the resonant tunneling of electrons, a transport phenomenon as it is, in the weak coupling limit. For  $\epsilon \rightarrow 0$ , the peaks of  $T(E_F)$  locate at the eigenenergies  $E_{n,\mu}$  of the closed ring [18]. In the presence of the SO interaction and a weak Zeeman coupling, the transmission probability is plotted as a function of the incident energy in Fig. 4. Every two peaks, which are closest to each other, locate at a pair of splitted energy levels, which come from the Kramers doublet  $(\Psi_{n,\mu}, \Psi_{-n-1,-\mu})$  in the absence of Zeeman coupling. With the AB flux being zero, the energy splittings in all the splitted energy levels are the same. Here we use the first-order perturbation which gives the energy correction but doesn't change the eigenfunction. As shown in Ref. [17], those eigenstates of the closed ring, with increasing energy, have the spin orientations and current directions in a sequence of

$$\cdots, [(+, d), (-, u)], [(-, d), (+, u)], [(+, d), (-, u)], [(-, d), (+, u)], \cdots, \quad (27)$$

where  $(s_1, s_2)$  refers to a single quantum state with  $s_1 = +$  (anticlockwise) or  $-$  (clockwise) denoting the current direction and  $s_2 = u$  (up) or  $d$  (down) denoting the spin orientation, and  $[(s_1, s_2), (-s_1, -s_2)]$  refers to a pair of energy levels from the Kramers doublet. The eigenstate correspondence so identified for  $T$  leads to interesting resonance behavior, as depicted in Fig. 4. It is seen that when a small AB flux is added to distinguish the current directions, each two paired peaks are separated by a distance, which takes the larger or the smaller value alternatingly. The reason is already clear in the sequence (27). In essence, since the current direction determines the sign of energy shift caused by a small AB flux,



for  $[(-, d), (+, u)]$  the energy splitting due to the small AB flux enhances that first caused by the Zeeman coupling, while for  $[(+, d), (-, u)]$  the energy splitting due to the small AB flux cancels part of that first caused by the Zeeman coupling. We want to point out that the essential character of the above correspondence between the equilibrium and the transport properties can be quantitatively, but not be qualitatively, affected by the disorder or scattering potential in the ring as long as the single particle picture holds for electronic states. In particular, such correspondence, if experimentally verified or excluded in some specific ring, may serve to detect if the electron-electron interaction qualitatively alters the electronic states.

#### IV. CONCLUSION

We have studied the motion of electrons confined in the perfect ring in the presence of the cylindrically symmetric spin-orbit interaction and Zeeman coupling, and the magnetic flux. We have obtained the exact solution of the closed ring by using the AA phase approach in which the geometric and the dynamical phases can be explicitly identified for all energy eigenstates. Starting from the exact solution for the closed ring, we have investigated the transport properties of the ring connected to current leads, with emphasis on the roles of the TRS and its breaking therein. From the derivation of the transmission probability, we have shown that in the presence of the TRS-breaking Zeeman coupling, it is physically impossible to adopt the picture that the spin precession of electrons encircling the ring results in some effective, spin-dependent Aharonov-Bohm flux in interference, thereby revealing the origin of the mistakes in some prior works. We have provided the numerical results for illustrating the joint effects of spin-orbit interaction, Zeeman coupling and magnetic flux. From the resonance behavior of the transmission probability in the weak coupling limit, we have found the observable correspondence between the TRS-breaking effects manifested by the persistent current and by the transmission probability as long as the single particle picture of electronic states holds.

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## FIGURES

FIG. 1. Schematic representation of the electronic waves propagating through the ring connected to current leads. The right junction is located at  $\theta = 0$  and the left junction at  $\theta = \pi$  with the upper branch lying within  $(0, \pi)$  and the lower branch within  $(\pi, 2\pi)$ .

FIG. 2. Transmission probability as a function of the AB flux for  $\epsilon = 0.25$ ,  $ka = 60.239$ ,  $a = 1\mu\text{m}$ , and  $\chi_2 = \frac{\pi}{6}$ . The dotted and the solid lines are associated with the absence of and the presence of the SO interaction of  $\alpha = 1.8$  respectively.

FIG. 3. Transmission probability as a function of the energy of incident electrons ( $E_F = \hbar^2 k^2 / 2m$ ), for  $\epsilon = 0.25$ ,  $a = 1\mu\text{m}$ ,  $\alpha = 1.8$ ,  $\chi_2 = \frac{\pi}{6}$ , and  $\phi = 0$ . The dotted line corresponds to the absence of SO interaction and Zeeman coupling, the solid line corresponds to the presence of SO interaction only, and the dashed-dotted line corresponds to the presence of both the SO interaction and the Zeeman coupling of  $B = 30\text{Gauss}$ .

FIG. 4. Transmission probability as a function of the energy of incident electrons ( $E_F = \hbar^2 k^2 / 2m$ ), for  $\epsilon = 0.005$ ,  $a = 1\mu\text{m}$ ,  $\alpha = 1.8$  and  $\chi_2 = \frac{\pi}{6}$ . The solid line corresponds to the presence of Zeeman coupling of  $B = 15\text{Gauss}$ , and the dashed-dotted line corresponds to the presence of the same Zeeman coupling and a magnetic flux of  $\phi = 0.02$ . a) Constant and alternating distances between paired peaks vs. energy, represented by the solid and the dashed-dotted lines respectively. b) Taken from a) for a clear illustration of the effect caused by  $\phi = 0.02$ .

Fig. 1

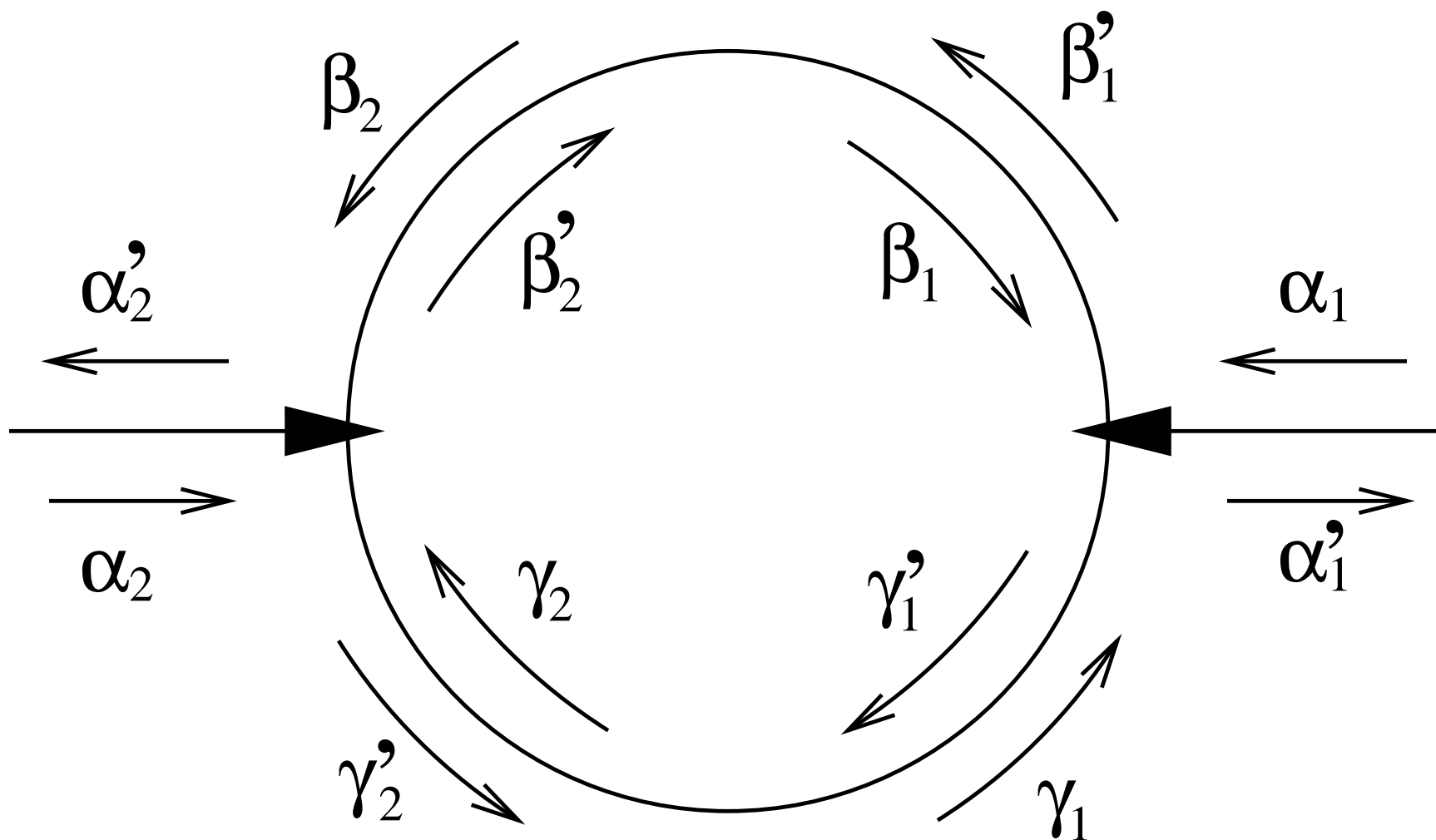


FIG.2

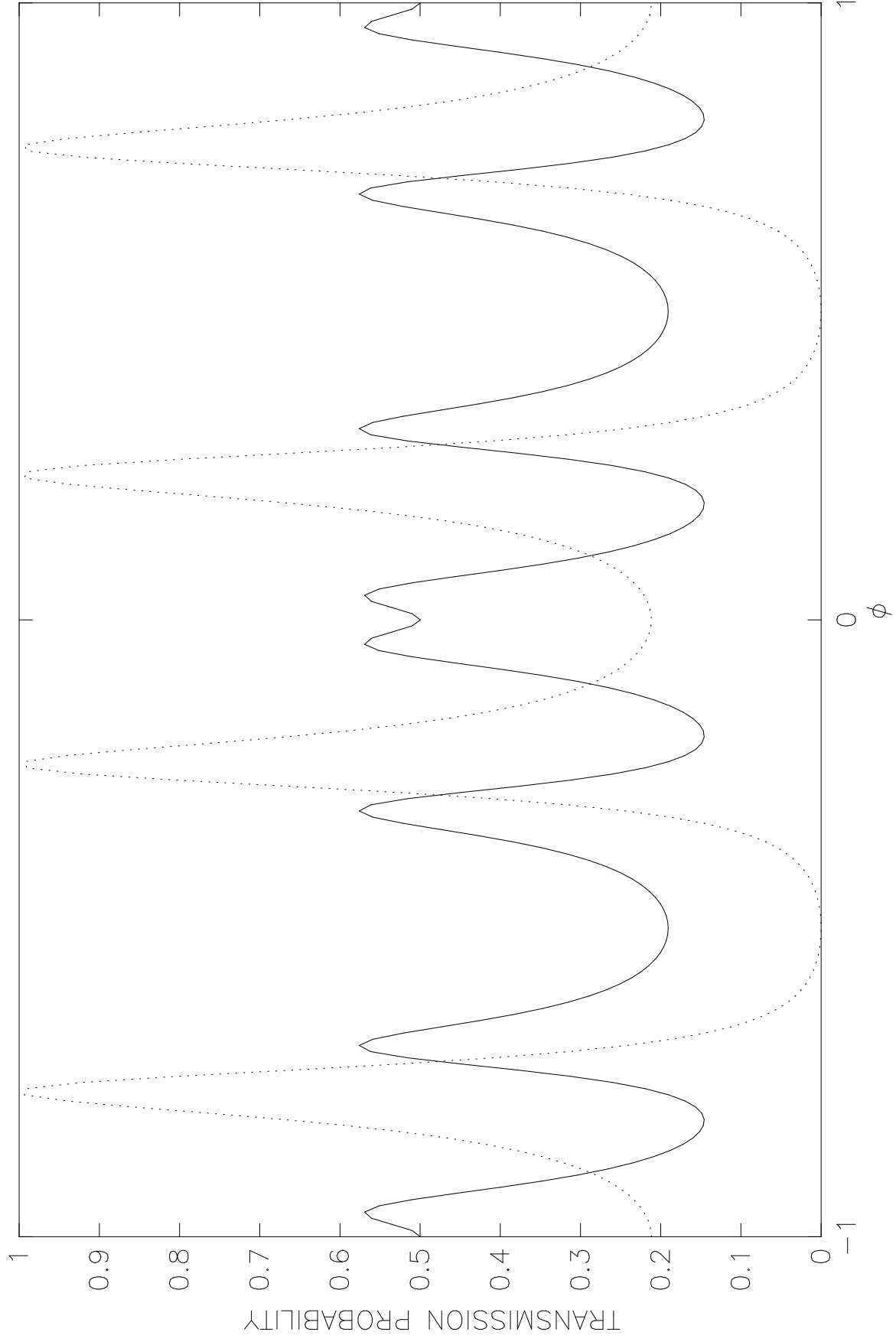


FIG.3

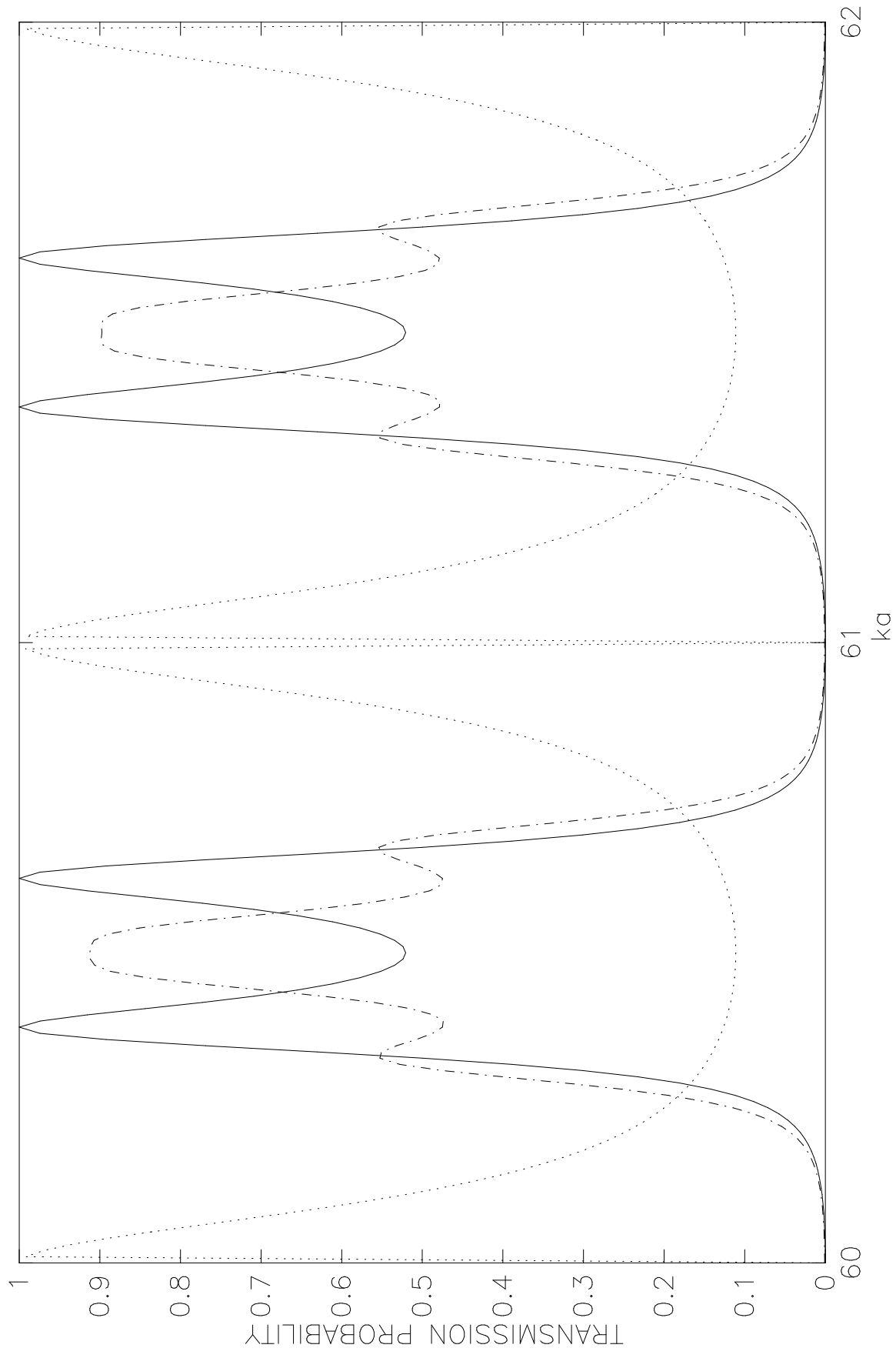


FIG.4

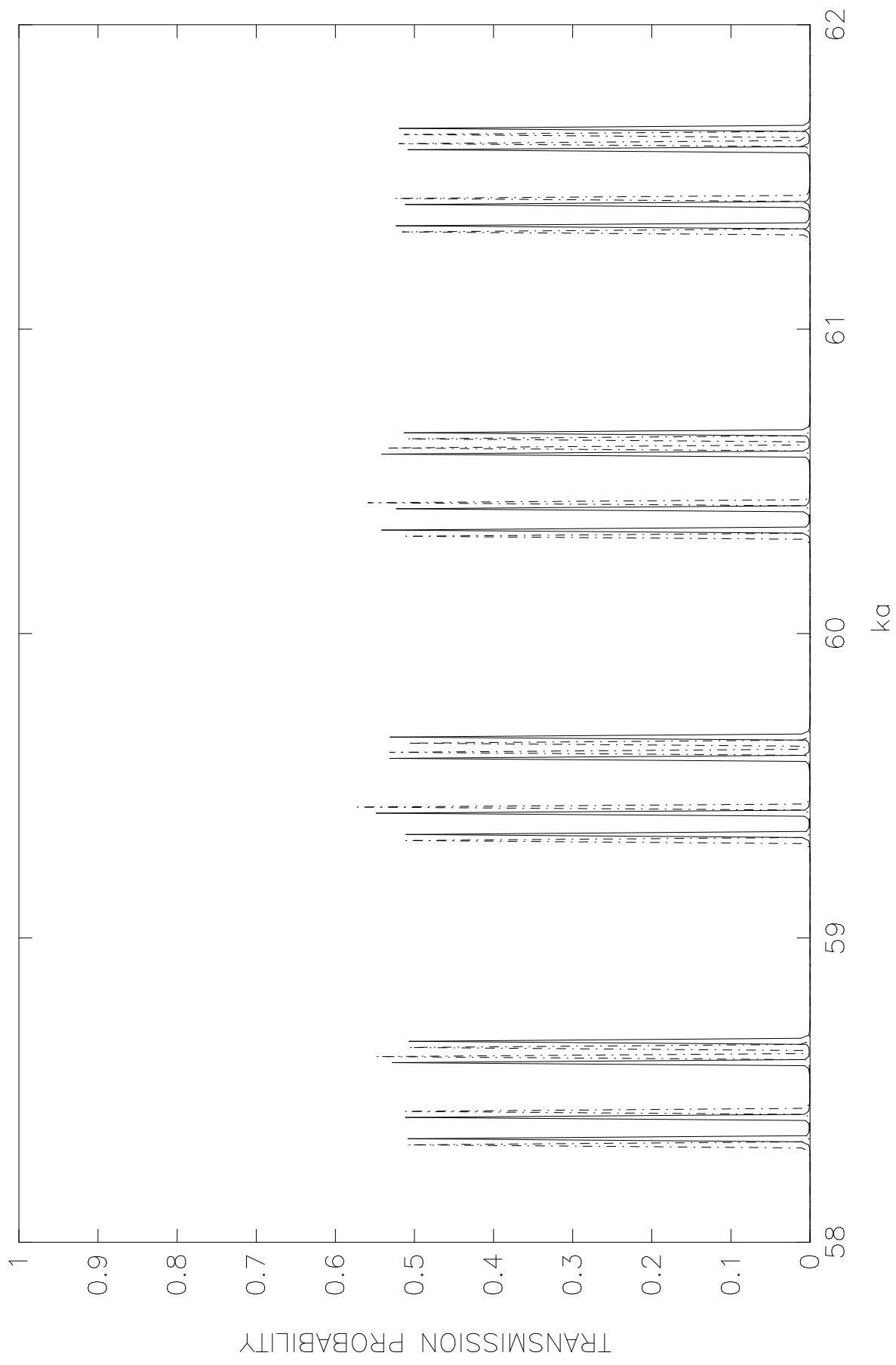




FIG.5

